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Comment on ‘Folding with thermal-mechanical feedback’

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“*Deeper physical insight combined with theoretical simplicity provides the short-cuts leading immediately to the core of extremely complex problems and to straightforward solutions. This cannot be achieved by methods which are sophisticated and ponderous even in simple cases. The process of thought which is involved here may be described as “cutting through the scientific red tape” and bypassing the slow grinding mills of formal scientific knowledge. Of course, formal knowledge is essential but, as for everything in life, the truth involves a matter of balance*”.

M. A. Biot
Acceptance speech, 1962 Timoshenko Medal

1. Introduction

In their recent paper, Hobbs et al. (2008) claim that (1) effective viscosity ratios and power-law stress exponents for natural rocks, as extrapolated from experimental conditions to those appropriate for the middle to lower crust, are typically too low for folds of finite amplitude to form according to the “traditional” buckle fold theory, referred to by them as the *“Biot process”*; (2) the geometry of natural folds, in particular the rather irregular fold form and the limited range in arclength/layer thickness ratios, cannot be predicted from this theory; and (3) a coupled model of folding with thermal-mechanical feedback is more appropriate for explaining natural folds. We strongly disagree with all these statements and show that correct application of the analytical buckling theory and its extension to finite amplitude using numerical models based on the same mechanical principles can explain the range of natural fold geometries. This approach provides relatively simple and clear insights into the fundamental processes governing such fold development. Our primary aim in this comment is to demonstrate that the “traditional” theory is appropriate for explaining natural examples. We therefore limit our discussion of the *“Biot process”* to interfacial viscous buckling (IVB) in the absence of elastic effects, as in the original work of Biot (1957, 1961, 1964, 1965), Sherwin and Chapple (1968), Fletcher (1974, 1977) and Smith (1975, 1977). However, for us the *“Biot process”* encompasses all aspects of the mechanical folding instability, such as multilayer systems, non-linear rheology, and finite strain effects.

2. Model of single-layer folding with thermal-mechanical feedback

Hobbs et al. (2008) discredit the *“Biot process”* as a viable folding mechanism without providing any sound counterarguments. They show no natural fold or any comparison between natural examples and numerical models that would illustrate any apparent problem with this approach. It is not sufficient to argue that low effective viscosity ratio between pairs of rocks may occur in nature – of course they do, and in this case buckle folds of finite amplitude simply do not develop (although passive amplification of irregularities can still occur in high strain zones). The necessary argument is to give examples of natural folds that have formed even when it can be reliably established that the viscosity ratio was too low for viscous buckling to have occurred. Such examples are not presented in the paper of Hobbs et al. (2008).

Hobbs et al. (2008) cite the absence of parasitic folds in the *“Biot process”* as an argument. However, there are at least five ways in which purely mechanical folding can produce parasitic folds: 1) at large strains a fold train starts to behave as an effectively thicker layer and larger wavelength folds develop, 2) the effective layer to matrix viscosity ratio (R) changes due to ambient temperature changes, 3) thickness (Frehner and Schmalholz, 2006) or 4) viscosity (Ramberg, 1964) variations in multilayer systems or 5)

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matrix anisotropy (Kocher et al., 2008) cause the simultaneous development of several wavelengths.

Hobbs et al. (2008) also claim that the type of boundary condition has a crucial influence on fold development in the *"Biot process"*. Clearly, for the linear viscous case this is incorrect as the development of fold amplitude with respect to bulk strain will always be the same, irrespective of the type or magnitude of the boundary conditions. They go as far as stating that *"for constant velocity and strain rate conditions amplification rates are relatively small and realistic folds do not develop unless viscosity ratios are of the order of 3000"*. This contradicts basically all previous analogue and numerical folding research, where folds were readily developed for much lower viscosity ratios for the given boundary conditions. It is even more puzzling because their own previous (Zhang et al., 2000) and current work also shows this, as can be seen from their Fig. 3b for $R = 20$ folds. Note though that the *"sensitivity analysis"* presented in their Fig. 3 aims at illustrating viscous folding, but a quick analysis (cf. Schmalholz and Podladchikov, 1999) reveals that elastic effects are strong for these runs (especially e to f). Also, what they call a *"perturbation velocity field"* is misleading, as it is usually termed the "total velocity field" (e.g. Cobbold, 1976; Passchier et al., 2005).

The alternative model presented by Hobbs et al. (2008) and the specific numerical model of single-layer folding are not appropriate either for the claimed middle to lower crustal conditions or for explaining the micro- to mesoscale folds directly observed by field geologists. They present a single-layer example for the case of thermal-mechanical feedback, in which a 400 m layer of *"feldspathic rock"* embedded in quartzite is shortened at temperatures of 510 K to 570 K (240 °C – 300 °C). The resulting large scale structures would usually be termed "pop-up structures" and the "*fine scale crenulations*" referred to as a mesh sensitive phenomenon. In fact, the "feldspathic rock" is the aplite whose creep parameters were reported in an abstract by Shelton and Tullis (1981) and the quartzite flow law is that of Hirth et al. (2001). From the range of feldspar-rich rock and quartzite flow laws available, this aplite is the weakest and this quartzite one of the strongest. The aplite – quartzite R varies between 2 and 1.2 over the model temperature range, and IVB is negligible, as the authors intend, to highlight the effects of thermal-mechanical instability. The model scale is $\sim 10^2$–10^5 times larger than that at which folds are commonly observed at outcrop, hand specimen or thin section scale (Fig. 1). The temperature range of 510 K to 570 K (240 °C–300 °C) is comparable to that of the brittle-ductile transition. It is not what is expected under middle to lower crustal conditions and is also rather low for crystal-plastic flow of quartzite and aplite. The effects of thermal-mechanical feedback are indeed significant at the model scale for the chosen rock properties and rate of shortening, at which the Peclet Number, written here as $Pe = W^2|D_{xx}|/\kappa = 0.9$, where initial model thickness is $W = 3000$ m, rate of shortening is the large value $|D_{xx}| = 10^{-13}\mathrm{s}^{-1}$ and thermal conductivity is $\kappa = 10^{-6}\ \mathrm{m}^2\ \mathrm{s}^{-1}$. For the single-layer folds shown in their Fig. 6, they report $|D_{xx}| = 10^{-15}\mathrm{s}^{-1}$, but this yields $Pe \sim 0.009$, contrary to the reported value. The use of W rather than layer thickness H is problematic when considering the case of a layer effectively embedded in an infinite medium. Scaling accordingly for a layer of thickness $H = 4$ cm, which is greater than that of any of the layers shown in Fig. 1, yields $Pe \sim 10^{-8}$, and the effect of thermal-mechanical feedback is negligible, as the authors themselves point out. Thus, unless a thermal-mechanical feedback associated with "small-scale compositional – fabric heterogeneities" produces regular folding in single layers in which mean fold arclength scales with layer thickness, a result neither adequately examined in their paper nor intuitively plausible, the authors, discounting IVB, leave us with no viable mechanism for the folds commonly observed in nature!

We now show that, with proper implementation of IVB for power-law layer and matrix pairs and with plausible creep parameters, folds will develop with a scale and geometry directly comparable to those formed in the middle and lower crust.

3. Arclength/thickness ratios for natural folds

Hobbs et al. (2008) assert that the mean fold arclength to thickness ratio (F_R) of natural single-layer folds is so small as to imply that R is too low for the folds to have initiated via IVB. They estimate R inappropriately, by equating F_R with the dominant wavelength to thickness ratio L_d/H, rather than the most-amplified value L_p/H, which takes into account the basic-state layer-parallel shortening. For L_d/H, they use, also inappropriately, the thin-plate approximation (e.g. Biot, 1961). In the limit of $R = 1$, $L_d/H = 2\pi$ for linear viscous media, and use of the thin-plate equation to estimate

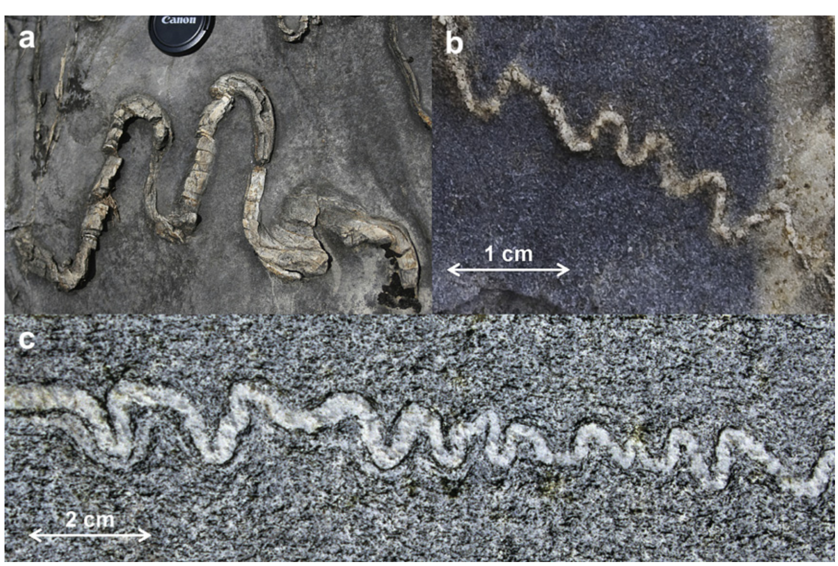

Fig. 1. Single-layer folds: (a) calc-silicate layer in coarse-grained calcite marble, Adamello, Italy; (b) pegmatitic quartz-feldspar layer in coarse-grained calcite marble, Adamello, Italy; and (c) pegmatitic quartz-feldspar layer in quartz-feldspar-biotite gneiss, Roveredo, Switzerland.

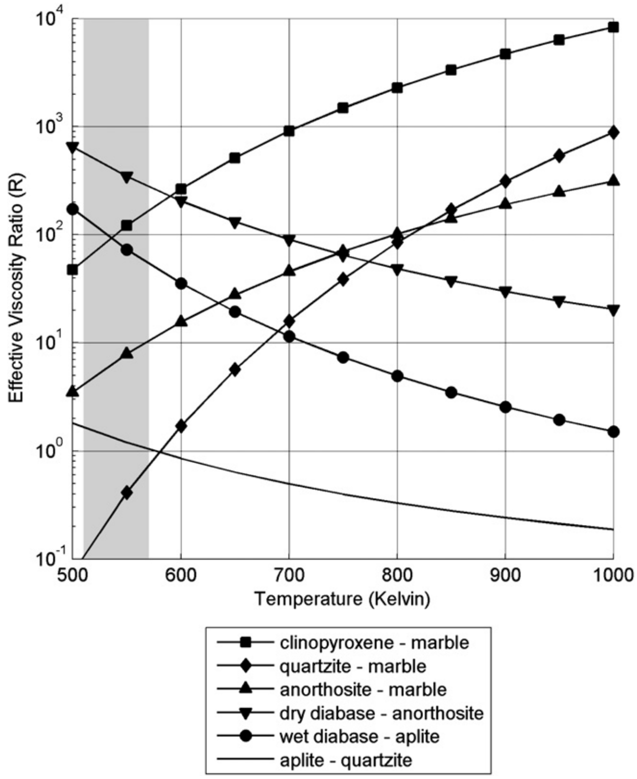

Fig. 2. Effective viscosity ratio as a function of temperature for six layer-matrix pairs at $|D_{xx}| = 10^{-14}s^{-1}$. T covers the typical range from the brittle-ductile transition to the base of the crust; the range considered by Hobbs et al. (2008) is underlain in gray.

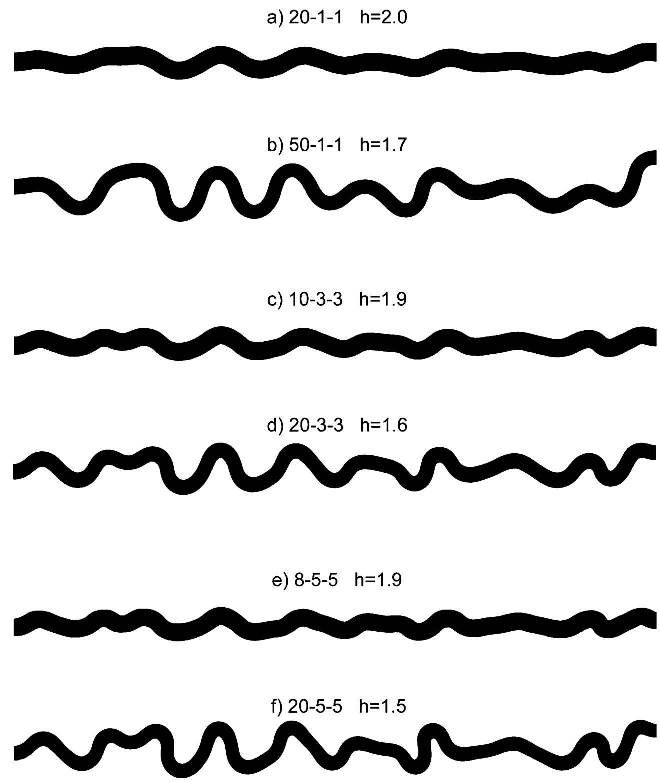

Fig. 3. Single-layer numerical IVB simulations with free-slip lateral boundary conditions, for the same initial random perturbation, for pure shear shortening of 50%. The code is: R-n_{matrix}-n_{layer}. h is the average ratio of final to original layer thickness.

viscosity ratios on the assumption that observed values of fold arclength to thickness are 2–7 (their p. 1575) is meaningless. At low R, thick-plate results must be used (Fletcher, 1974, 1977; Smith, 1975, 1977, 1979). Since fold trains are not mathematically periodic, it is misleading to talk of fold arclength as "wavelength" and F_R is used to estimate the most-amplified value L_P/H (Fletcher and Sherwin, 1978).

Reported values obtained for F_R are 4.0, 4.5, 5.1, 5.2, 5.5 and 6.8 for quartz veins or quartzite layers in slate or phyllite matrix (Sherwin and Chapple, 1968), 6.7, 9.8, 12.1, and 15.3 for quartz veins in mafic, psammitic, and pelitic schists (Shimamoto and Hara, 1976), 6.5 and 7.1 for limestone in slate (Hudleston and Holst, 1984), and 9.4 for limestone in shale (Fletcher, 1974). The single-layer folds in Fig. 1 have (a)$F_R \cong 12$, (b)$F_R \cong 6.9$, and (c) $F_R \cong 5.9$. In summary, these measurements show that F_R ranges from 4 to 15, with a median of 6.7. Many examples have stiff layers composed of quartzite, vein quartz, aplite, pegmatite or limestone, for which laboratory experiments yield power-law stress exponents (n) of $\sim$3–4 for quartzite and $\geq$4 for coarse calcite (e.g. see compilations in Brodie and Rutter, 2000; Evans and Kohlstedt, 1995; Kirby and Kronenberg, 1987).

Using $F_R = L_P/H$, stress exponent $n = 3$ for layer and matrix, and requiring a maximum amplification $A_{max} = 50$ to achieve selectivity appropriate to natural single-layer folds, the thick-plate result, which accounts for uniform layer-parallel shortening in the low limb-dip phase of wavelength selection, provides estimates of both R and the layer-parallel shortening. This is a variant of what authors of the above-mentioned papers, including Sherwin and Chapple (1968), did to estimate R. For $L_P/H = 4$, 5, 6, 7, 10, 12 and 15, we obtain $R = 6.5$, 10, 14, 21, 51, 78 and 150 and the corresponding layer-parallel shortenings of 43, 33, 19, 15, 12, 8 and 6%. We now show that such viscosity ratios are attained for representative rock pairs at temperatures (T) and rates of shortening (D_{xx}) typical of the middle to lower crust.

4. Viscosity ratio for rock pairs

In Fig. 2 R is plotted as a function of T for six layer-matrix pairs and $|D_{xx}| = 10^{-14}s^{-1}$. The additional flow laws (aplite and quartzite are discussed above) are clinopyroxene and anorthosite (Kirby and Kronenberg, 1987), marble (Walker et al., 1990, 1 mm grain size), dry diabase (Mackwell et al., 1998, Columbia) , and wet diabase (Shelton and Tullis, 1981). Note, that the stiff layer – marble pairs show R *increasing* with temperature, which is so if $(Q/n)_{layer} - (Q/n)_{matrix} < 0$, where Q is the activation energy of the Arrhenius temperature dependence. Hobbs et al. (2008) use the aplite – quartzite pair in their single-layer example, which yields $R \leq 2$ over the range 510 K–570 K they consider. Their wet diabase–aplite pair exhibits viscosity ratios in the range 500 K–700 K that are still adequately large to produce IVB. Other suitable rock pairs in the middle to lower crust exhibit much larger ratios. The clinopyroxene - marble pair is applicable to the natural example in Fig. 1a, the quartzite-marble and anorthosite-marble pairs to that in Fig. 1b. The dry diabase - anorthosite is representative of the granulite facies, typical for the lower crust. Hobbs et al. (2008) view pairs such as quartzite-schist or quartzite-marble, to be *"exceptions"*. However, these pairs are common in nature and several of the F_R values reported above were obtained for them. The authors' aplite-quartzite pair is rather the "exception."

5. Numerical models of viscous folding

Numerical and analogue models provide a definitive test of whether folding with specified rheological parameters will occur.

The simulations of single-layer folding (Fig. 3) were obtained using a modified version of the FEM code MILAMIN (Dabrowski et al., 2008), where only the uncoupled mechanical problem is solved. Horizontal velocity at the lateral, free-slip boundaries is adjusted to give constant mean strain rate – the case for which Hobbs et al. (2008) claim there will be no realistic fold development unless $R \geq 3000$. Vertical boundaries are sufficiently distant from the layer to not affect fold growth. The same initial random perturbation (red noise) is used in all models, with the maximum vertical variation limited to 1/20 of the layer thickness.

Folds formed in all runs, for R as low as 8 (Fig. 3e). This is consistent with, for example, the paraffin wax analogue experiments of Cobbold (1975), where folds formed in a 10-2.6-2.6 model. $R \geq 20$ was required to generate folds in the linear viscous case (Fig. 3a). The folds have the moderate regularity seen in natural counterparts, in disagreement with the statement of Hobbs et al. (2008) that *"for purely viscous materials, periodic fold geometries result from the classical theories no matter what boundary conditions, initial geometry or initial deviations from the ideal planar state exist"*. The rationale for limited regularity is discussed in Biot (1961). The growth-rate spectrum has a single maximum and a bandwidth that is not particularly narrow for single-layer folding. At low viscosity ratio (Hudleston, 1973; Ramberg, 1970; Sherwin and Chapple, 1968), layer thickening prior to finite amplitude buckling is substantial and F_R values as low as 4 result (Fig. 3e). Our simulations show that IVB can yield small F_R and typical irregular fold trains, as in Fig. 1, without recourse to thermal-mechanical feedback. Since the model has no intrinsic length, it is scale independent. IVB is therefore applicable to folds at the observed mm–m scale, in direct contrast to the model of Hobbs et al. (2008).

6. Conclusions

Based on the thick-plate buckling theory for power-law rock materials, $R \geq 5$–10 is required to obtain the range $F_R \sim 4$–15 of natural single-layer folds. Extrapolation of laboratory creep data shows that such values of R are attained or exceeded for a wide range of rock pairs at conditions appropriate to deformation in the middle to lower crust. Using finite-elements models of buckle folds without thermal-mechanical feedback (Fig. 3), we demonstrate that realistic single-layer fold geometries with values of F_R as small as 4 are developed for viscosity ratios on the order of 8–20 in power-law material with stress exponents of 3–5. The numerical models are in accord with analytical models describing the initiation and growth of buckle folds in single layers, without recourse to thermal-mechanical or other feedback processes. The models with thermal-mechanical feedback presented by Hobbs et al. (2008) are not capable of driving folding at the smaller scale of typical field observation, whereas the "traditional" interfacial viscous buckling is perfectly appropriate.

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